

<span id="page-1-0"></span>

### **2.1 Model definition**

<span id="page-2-1"></span><span id="page-2-0"></span>
$$
\begin{aligned}\n\ddot{\mathbf{F}}_{\mathbf{A}} & \mathbf{a} & \mathbf{b} & \mathbf{b} & \mathbf{b} & \mathbf{c} & \mathbf{c}
$$



### **2.2 Single bump solutions**

<span id="page-3-0"></span>

<span id="page-4-0"></span>
$$
-30 \qquad -15 \qquad 0 \qquad 15 \qquad 30
$$

<span id="page-5-0"></span>
$$
\begin{aligned}\n\hat{J} &= 1, \quad \hat{J} &= 1, \quad \hat{J} &= \hat{J} \quad \hat{
$$









<span id="page-11-0"></span>

 $B_{\rm B}$  boundary, the stored angles repeating  $\widetilde{\mathbf{F}}$  , the stored angles repeating  $\widetilde{\mathbf{F}}$ There is an abrupt transition in the MSE corresponding to this boundary point  $\left(\begin{array}{cc} 0 & t \end{array}\right)$  $\left(\begin{array}{cc} 0 & t \end{array}\right)$  $\left(\begin{array}{cc} 0 & t \end{array}\right)$ , Importantly, the MSE is  $\mathcal{A}$ limited from below at each <sup>0</sup> by the variance a single bump  $\Delta = D \cdot T$ ,  $\Delta = \frac{1}{2}$ even though the peak MSE grows significantly for the case  $\mathcal{A} = 10, \dots, \dots, \dots, \dots, \dots, \dots$  $s_{\rm F}$ significantly smaller at large values of  $\Gamma_{\rm eff}$  , since individualler  $\tilde{\Gamma}_{\rm eff}$  , since individualle  $\mathbf{d}(\cdot, \cdot, 4)$ .  $\mathcal{T} \subset \mathbb{R}$  bound on the MSE produced by a single bump's trajectory is approached when the two bumps are  $f$ initiated at the same location of the same location  $( \begin{array}{cccc} 0 & = & 0), & \ldots \end{array}$  , when the theorem وأوس المعارفين والأستريط والمتحدث والمستعمل والمستعمر المستحر والمستحر والمستحر والمستحر والمستحر  $t$ there will always be vanishingly small republic that  $\mathcal{A}_\text{c}$  that  $\mathcal{A}_\text{c}$  that  $\mathcal{A}_\text{c}$ will tend to push bumps farther apart, we see that the event farther apart,  $\mathbb{P}^1$ for <sup>0</sup> 6, the MSE appears to approach a lower limit. تعرض المعادلة وأنسعت والمحاربة أنست والمحارب أنساق بالعام والمحارب are only one of  $\mathbb{R}^{d-1}$  and  $\mathbb{R}^{d-1}$  and  $\mathbb{R}^{d}$  . We have  $\mathbb{R}^{d}$ where  $\mathcal{J}_1$  are smaller  $\mathcal{J}_2$  . These effects are smaller than  $\mathcal{J}_2$  are smaller than  $\mathcal{J}_3$ لی میگر از موسیق گوند از برگ ویک سال گارگی و درگر ویک گو our numerical integration scheme, so we would expect the source  $\mathcal{T}_1$  $s_{\rm c}$ strength of repulsion to be weaker than the pinning produced than the pinning produced produced produced produced produced by  $\sigma$  $(0.005).$ Performance on the two-item WM task with random initial targets *φ*<sup>1</sup> and *φ* is considered in Fig. [e](#page-11-0). Recall variability, represented by the MSE, is greater than what when when would be predicted by a model that allows distinct slots distinct slots  $\mathcal{P}_\text{c}$  $f_{\rm c}$  , there  $f_{\rm c}$  item. Note, there have been efforts to revise there is revise to revise the revise the revise the revise to revise the revise to revise the revise to revise the revise to revise the revise the re slot model (Zhang and Luck [2008;](#page-22-5) Cowan  $\sim$  010), so that error increases when considering two items versus one item. However, the increases in error arising from neural activity dynamics of the function of the much more nuanced than  $\mathcal{A}$ would be possible for previous phenomenological slots phenomenological slots phenomenological slots  $\mathcal{P}_\text{max}$ or resources models (Zhang and Luck [2008;](#page-22-5) Bays and Husain  $(00)$ . Both items (bumps) are stored in a single in  $\mathbb{R}^4$  $\vec{J}$ network, producing interactions between bumps when items when items when items when items when items when items  $\vec{J}$ are in interesting contributes and  $\mathbb{P}_\alpha$  contributes and additional sources and  $\mathbb{P}_\alpha$ of variability to the recall  $\mathcal{A}$  of  $\mathcal{A}$  systematic produces a systematic produces a systematic produces a systematic produces  $\mathcal{A}$ shift in the remembered location of items, as does repelling. The frequency of these interactions grows as the syngaptic strength parameter  $\mathcal{A}_\text{in}$  increased, counteraction  $\mathcal{A}$  the reduction in diffusion also produced by increasing  $\mathcal{A}.$ This tradeoff produces a non-monotonic dependence of the  $\mathcal{A}_{\blacktriangledown_{\mathcal{A}}}, \ \mathcal{A}$  $\mathcal{A}_{\blacktriangledown_{\mathcal{A}}}, \ \mathcal{A}$  $\mathcal{A}_{\blacktriangledown_{\mathcal{A}}}, \ \mathcal{A}$  (Fig. e), so the increase is an optimal  $\mathcal{A}_{\text{c}}$  or the item  $s_{\rm tot}$  and  $\mathcal{F}^{\dagger}$  and  $\mathbf{v}_{\rm tot}$  and  $\mathbf{v}_{\rm tot}$  and probability and low-probability and low-probability and  $\mathbf{v}_{\rm tot}$ of bundles bundles bundles when  $\mathbf{1}_{\{1,2\}}$  and  $\mathbf{1}_{\{2,3\}}$  and  $\mathbf{1}_{\{3,4\}}$ so even though the peak MSE is much larger than for the [c](#page-11-0)ases  $A = 1, \ldots, \ldots$ ,  $\sigma$  and  $\sigma$  average  $M$ since bumps are less susceptible to stochastic perturbations.  $\mathcal{H}$  , the interaction of interaction bumps model is larger than  $\mathcal{H}$  and  $\mathcal{H}$ would be predicted by a slot model that assumes MSE is assumed that assumes  $\mathcal{M}_\text{S}$  is assumed to the assumes MSE is assumed to the assume of the assumed to the assume of the assumed to the assumed to the assumed to unchanged as the number of items is increased up to some  $f(x) = \left(\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}\right)$  $f(x) = \left(\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}\right)$  $f(x) = \left(\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}\right)$ .



### **4 Multiple interacting bumps**

Recent models of multi-item WM focus on uncovering the nature of item-number limitations, as they impact response 014). be altered to capture errors that either reflect a finite capacity or the distribution of  $f_{\lambda}$  (Zhang and Luck [2008\)](#page-22-5), but physiologically-inspired models account for the archi $t$ et underlying dynamics of neural circuits  $\mathcal{M}_{\bullet}$ (Bays [2015\)](#page-21-14). The work of Edin et al. [\(2009\)](#page-21-3), Weight of Edin et al. (2015).  $e_t$  [\(2012\)](#page-22-3), and Almeida et al. [\(2015\)](#page-21-4). recurrent spiking network can support multiple bumps that each individually encode a different item. Our model is a tractable version of these previous studies, allowing us to derive explicit expressions describing limitations of the network. Prior to developing effective equations for bump inter $f_{\rm c}$  , we consider the problem of network capacity. This capacity is network capacity. This capacity is network capacity. is one way in which our model differs from the standard resource model of WM. Only a finite number of bumps can be stored in the recurrent network, and this upper limit is  $d\vec{r}$ determined by the choice of the synaptic strength parameter  $\vec{r}$  ${\cal A}.$  However, we note that we note that upper limit is quite large. We are can approximate this limit by again examining a stationary solution problem.

### **4.1 Network capacity**

 $\widetilde{\Theta}$  in  $\widetilde{\Theta}$  is  $\widetilde{\Theta}$  in the problem of identifying network capacity by  $\widetilde{\Theta}$ attempting to identify multi-bump stationary stations to identify  $\epsilon$  $\left( \begin{array}{c} 1,1,1,1 \end{array} \right)$  in the absence of  $\left( \begin{array}{cc} 1,1,1,1 \end{array} \right)$ solutions are not stable in the limit *L* → ∞ (Laing and Troy  $(00\ \mathrm{s})$ , since  $\mathbb{R}$  and  $\mathbb{R}$  regions  $\mathbb{R}$   $\mathbb{R}$  regions  $\mathbb{R}$   $\mathbb{R}$  repulsive  $\mathbb{R}$ drift on one another. If bumps are spaced evenly around the domain, the conformation is stable since the repulsive forces acting on each bump from either direction balance. Thus, stable multi-bump solutions constitute a periodic patient in the second patient of the second patient of  $\mathbf{f}_\alpha$  , wraps around that wraps around the domain. One question is just how is just how  $\alpha$ the minimal period of this pattern changes as the syn $t$  this pattern changes as the synaptic s  $\mathcal{A}_i$  is changed. Since  $\mathcal{A}_i$  in ite $\mathcal{A}_i$  and  $\mathcal{A}_i$  and  $\mathcal{A}_i$  apacity  $\mathcal{A}_i$ A



# $\hat{H}$   $\hat{H$

## $\mathcal{L}_{\bullet}$ . ( $\mathcal{A}$



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1*,* ...*, M*). For simplicity, we focus on the case of two symmetrically-placed bumps in two distinct layers *A(*0*)* = [−*b*0*,* −*a*0]×{*j* }∪[*a*0*, b*0]×{*k*}*,* of a symmetric network (*Wjk(x) Wkj (x)*, *j,k*). Therefore, we can write Eq. [\(5.5a, b\)](#page-17-0) as *<sup>a</sup>*1*(t)* <sup>=</sup> <sup>1</sup> *<sup>α</sup>*¯ *<sup>θ</sup>* <sup>−</sup> *W (b*<sup>1</sup> <sup>−</sup> *<sup>a</sup>*1*)* <sup>−</sup> *Wjk(b* <sup>−</sup> *<sup>a</sup>*1*)* +*Wjk(a* − *a*1*) ,* (5.6a) *<sup>b</sup>*1*(t)* = − <sup>1</sup> *<sup>α</sup>*¯ *<sup>θ</sup>* <sup>−</sup> *W (b*<sup>1</sup> <sup>−</sup> *<sup>a</sup>*1*)* <sup>−</sup> *Wjk(b* <sup>−</sup> *<sup>b</sup>*1*)* +*Wjk(a* − *b*1*) ,* (5.6b) *<sup>a</sup> (t)* <sup>=</sup> <sup>1</sup> *<sup>α</sup>*¯ *<sup>θ</sup>* <sup>−</sup> *W (b* <sup>−</sup> *<sup>a</sup> )* <sup>−</sup> *Wkj*

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## the interfaces can be obtained by evolving an integral equation describing the dynamically evolving gradient at the

**Acknowledgements: الجمع المجموع المجموع** Directions in Data Discovery in Undergraduate Education (NSF DMS-1407340).  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  ( $\frac{1}{2}$ ).

#### **Compliance with Ethical Standards**

**Conflict of interests** of the authors declare that the declare that they have no conflict of interests of the set interest.

### <span id="page-21-12"></span>**References**

- <span id="page-21-4"></span><span id="page-21-3"></span> $A_1$ meida, R., Barbosa, J., Barbosa, A. (2015). Neural circuit basis b of visuo-spatial working memory precision: a computational and  $\mathcal{F}_{\lambda}$ , **Jo** rnal of Ne<sub>r</sub> rophysiology, 114(3), 1806–1818. Amari $(1-\varepsilon)$  , and the pattern formation in lateral in lateral in lateral in lateral in lateral in lateral in  $\Box$  **1**  $\Box$  **1**  $\Box$  *Biological C<sub>r</sub> bernetics, 27( ),*
- <span id="page-21-9"></span><span id="page-21-5"></span>Amari, S. (2014). Heaviside world: excitation and self-organization of  $\sigma$  and self-organization of  $\sigma$  $\mathbb{R}$ <sub>1,  $\mathbb{R}$ </sub>,  $\mathbb{R}$ *ne* ral fields (see and 118). Springer.
- Avitabile, D., Desroches, M., Knobloch, E. (2017). Spatiotemporal
	- can be in  $\mathbb{R}^d$  in  $\mathbb{R}^d$  in  $\mathbb{R}^d$ , i.e.,  $\mathbb{R}^d$  *Physical Re<sup>v</sup>iew E*, 95(4),  $04$ ,  $05$ .
- <span id="page-21-2"></span><span id="page-21-1"></span>Barak, O.,  $\mathcal{R}$  Tsodyks, M. (2014). Working models of working  $\mathcal{R}$ **ri** *C* rrent Opinion in Neurobiolog<sub>3</sub>, 25, 0–4.
- Bays,  $\mathcal{W}_m$  (2014). Noise in neural populations accounts for example  $\mathcal{W}_m$  $\mu$ , **3** *Jo* rnal of Ne roscience, 34(10), 35. Bays,  $\mathcal{W}_{\bullet}$  , (1015). Spinkes not such slots: noise in neural populations limits: noise in neural populations limits:
- <span id="page-21-14"></span><span id="page-21-8"></span>*r J Trends in Cognitive Sciences, 19*(*),* 4 1 4 .  $B_1, \ldots, \mu, \&$  Husain,  $\mathcal{H}$  (2008). Dependence in limited working  $\mathcal{F}$ **memory resources in human vision.** Science,  $321(5 \ 0)$ , 51–54. Bays,  $\mathcal{A}_r$ ,  $\mathcal{A$
- <span id="page-21-13"></span>working memory is set by allocation of a shared resource. **J**ournal *of Vision*, 9(10), .  $\mathcal{F}_{\text{max}}$  ,  $\mathcal{F}_{\text{max}}$  (2005). We arrive that synaptically pulses in synaptically
- coupled neural media. *SIAM Journal on Applied Mathematics* , *66*(1), 5 1.
- $\beta$  .  $\beta$  (200). Stochastic neural field theory and the systemsize expansion. *SIAM Journal on Applied Mathematics*, *70*(5),  $14$  15 1.
- <span id="page-21-11"></span><span id="page-21-10"></span> $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ . Spatio $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ . Spation of continuum neural dynamics of conti *Jo* rnal of Ph sics A: Ma hema ical and Theore ical, 45(),  $,001$ ,  $,10$ .
- <span id="page-21-7"></span><span id="page-21-6"></span><span id="page-21-0"></span> $\mathcal{F}_1, \ldots, \ldots, \mathcal{R}_{n-1}$  .  $\ldots$  ,  $\ldots$  (2015). Nonelinear langevinne equations for wandering patterns in stochastic neural fields. *SIAM Jo rnal on Applied D namical S s ems, 14*(1), 05 4.
- $L_{\rm crit}$  ,  $L_{\rm crit}$  ,  $R_{\rm crit}$  ,  $L_{\rm crit}$  (2003a). Particle for non-local models for  $P$ *SIAM Jo rnal on Applied D namical S s ems, 2(), 4* 51.
- <span id="page-22-8"></span>Laing, C.R.,  $\&$   $\in$  ,  $\ldots$  ,  $\in$   $(00)$  b). The complete amari-field solutions of a type models of neuronal pattern formation. *Physica D: Nonlinear Phenomena*, 178(), 1 0 1.
- $L_{\rm B}$ ,  $\vec{\theta}$ ,  $\vec{\theta}$ ,  $\vec{\theta}$ ,  $L_{\rm B}$ ,  $L_{\rm B}$ ,  $L_{\rm B}$ ,  $L_{\rm B}$  (2002). Multiple bumps in a neuronal model of working memory. *SIAM Jo rnal on Applied Ma hema ics, 63*(1),  $\qquad \quad$ .
- <span id="page-22-0"></span> $L_1$ ,  $\mathbf{E}_1$ ,  $\mathbf{E}_2$ ,  $\mathbf{E}_2$ ,  $\mathbf{L}_3$ ,  $(01)$ . Capacity and precision in an  $\blacksquare$  *Jo* rnal of Vision,  $12(3)$ , 13–13.
- $L_1$ ,  $S$ ,  $\&$   $\cdots$ ,  $M_n$ . (2014). Balanced cortical microcircuitry corresponding corresponding corresponding to  $L_1$ . for spatial working memory based on corrective feedback control. *Jo* rnal of Ne roscience,  $34(0)$ ,  $0 \quad 0.$
- <span id="page-22-1"></span>Lu, Sato, Sato, Sato, Si, A.mari. (2011). The first and the second  $J_{\rm eff}$ collisions in a two-dimensional neural field. *Neural Computation*,  $23(5)$ , 14 <sup>1</sup> 0.
- <span id="page-22-3"></span>Luck, S.J.,  $\&$   $\alpha$ ,  $\vec{P}$ ,  $\ldots$  (1997). The capacity of visual working  $\vec{P}$ **i** and conjunctions. **Nature**,  $390(-5)$ , 279.
- Luck, S.J., & Vogel, E.K. (2013). Visual working memory capacity: from psychophysics and neurobiology to individual differences. *Trends in Cogni*  $i^*$ e Sciences,  $17(8)$ , 3400.
- <span id="page-22-7"></span> $\mathbf{M}_\bullet$  ,  $\mathbf{J}_\bullet$  ,  $\mathbf{N}_{\bullet,\circ,\bullet}$  ,  $\mathbf{M}_\bullet$  ,  $\mathbf{M}_\bullet$  (2014). Changing concepts of  $\mathbf{J}_\bullet$  ,  $\mathbf{M}_\bullet$  $\mathbb{N}$  *Na* re Ne roscience, 17( ), 4  $\pm$  5.
- <span id="page-22-6"></span><span id="page-22-5"></span><span id="page-22-4"></span><span id="page-22-2"></span> $\mathbf{A}_{\bullet}$  , Myser, J., Klingberg, J., Medicine, J., T., (2006). A biophysical of the  $\mathbf{A}_{\bullet}$ model of multiple-item working memory: a computational and  $n_1$ <sub>neuroimaging</sub> study. *Ne roscience*,