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Multistability in coupled oscillator systems with higher-order interactions and community structure

Special Collection: Dynamics on Networks with Higher-Order Interactions

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a hypergraph or a simplex when they are encoded in a simplicial complex).

For the network Kuramoto model with pairwise interactions, the role of network community structure is well studied as it gives rise to a variety of phenomena including hierarchical synchronization,[36] chimera states,[37] and nonmonotonic synchronization transitions.[38] Despite the prevalence of community structure in a wide range of real-world networks,[39,40] the effect of community structure in networks with higher-order interactions is not yet studied. In this paper, we study a generalization of the Kuramoto model that includes higher-order interactions and community structure. We apply the Ott–Antonsen ansatz[41,42] to obtain a low-dimensional system of equations. Focusing on the case of two communities, we find that, depending on the strength of community structure, the system admits multiple coexisting stable synchronization states. In addition to both communities being incoherent and both communities being synchronized and in-phase synchronized with one another, i.e., entrained with a macroscopic phase difference $= 0$, a handful of more complicated states exist. In one such state, one community is synchronized while the other is (nearly) incoherent. Another state is characterized by both communities being synchronized but each community is anti-phase synchronized with one another, i.e., forming two clusters with a phase difference $=$. Finally, when inter-community coupling is negative a surprising state emerges where both communities are synchronized but equation for , namely,

$$— = − + \frac{1}{2} − 2 . \tag{7}$$

To connect the dynamics of with those of the order param-

where we have used that $\tilde{}_{2}^{0} = 0$. Collecting terms at different orders of structure and higher-order interactions in less simplified scenarios will result in complex oscillation dynamics.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Per Sebastian Skardal: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Sabina Adhikari**: Writing – review & editing (equal). **Juan G. Restrepo**: Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX: STABILITY OF THE IN-PHASE AND ANTI-PHASE STATES

In this Appendix, we provide detailed calculations for the linear stability of the in-phase and anti-phase states discussed in Sec. IV.

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Comsd] -2 0.6 0 g g 10 0 0 10 0 0 cm BT /R64 7.5 Tf 1 0 0 1 254.652527513.117 [(C)10.409536(l)80.402046(o] /R59 7.5 Tf 1150586 0 Td [(.)-313.5.593(037113224.6.3(.))0.390346(on1808)0.390346(on]