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Network connectivity during mergers and growth: Optimizing the addition of a module

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The principal eigenvalue λ of a network's adjacency matrix often determines dynamics on the network (e.g., in synchronization and spreading processes) and some of its structural properties (e.g., robustness against failure or attack) and is therefore a good indicator for how "strongly" a network is connected. We study how λ is modified by the addition of a module, or community, which has broad applications, ranging from those involving a single

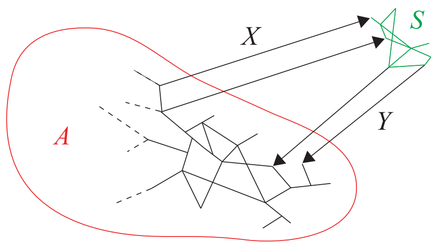
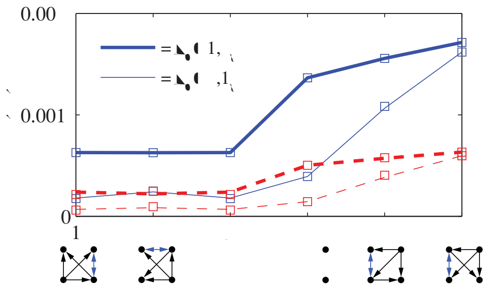


FIG. 1. (Color online) A module (described by matrix S) is connected to the original network (described by matrix A) using directed connections (described by the matrices X and Y).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successively maximizing the third-, fourth-, . . . , kth-order terms until all degrees of freedom have been exhausted. While this strategy of successive maximization does not guarantee the optimal connections (which would require considering all possible links between S and A), it is computationally efficient and ensures a near-optimal solution.

IV. DISCUSSION

While we have presented an efficient strategy for maximizing $\delta\lambda$

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