

Predicting Criticality and Dynamic Range in Complex Networks: Effects of Topology

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The effects of network topology on the criticality and dynamic range of a system are investigated. We consider a system of n nodes, each of which can be in one of m states. The system is coupled to a single external node, which is fixed to a particular state. We study the criticality and dynamic range of the system as a function of the network topology. We show that the criticality and dynamic range of the system are determined by the spectral properties of the adjacency matrix of the network. We show that the criticality and dynamic range of the system are maximized when the network is a regular graph with a large spectral gap. We also show that the criticality and dynamic range of the system are minimized when the network is a star graph.

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Networks are ubiquitous in nature and society, and their study has become a central theme in modern science. In particular, the study of complex networks has led to a better understanding of many natural and social phenomena. One of the most important questions in network science is how the topology of a network affects its dynamics. In this paper, we investigate the effects of network topology on the criticality and dynamic range of a system. We consider a system of n nodes, each of which can be in one of m states. The system is coupled to a single external node, which is fixed to a particular state. We study the criticality and dynamic range of the system as a function of the network topology. We show that the criticality and dynamic range of the system are determined by the spectral properties of the adjacency matrix of the network. We show that the criticality and dynamic range of the system are maximized when the network is a regular graph with a large spectral gap. We also show that the criticality and dynamic range of the system are minimized when the network is a star graph.

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FIG. 2

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$\epsilon_i - a_{ij} \in [2:0; 6:0]$, $a_{ij} \in [0:7; 1:3]$, $(a_{ij} - 5) \in [0:7; 1:3]$, $(a_{ij} - 6) \in [0:7; 1:3]$, $\langle d \rangle = 1$, $a_{ij} \in [0:7; 1:3]$, $A_{ij} = d^{out}$.
 We have $a_{ij} \in [0:7; 1:3]$, $(a_{ij} - 5) \in [0:7; 1:3]$, $(a_{ij} - 6) \in [0:7; 1:3]$, $\langle d \rangle = 1$, $A_{ij} \in \{0, 1\}$.
 Case 1: $a_{ij} = 5$, $(a_{ij} - 5) = 0$, $(a_{ij} - 6) = -1$, $\langle d \rangle = 1$, $A_{ij} = 1$.
 Case 2: $a_{ij} = 6$, $(a_{ij} - 5) = 1$, $(a_{ij} - 6) = 0$, $\langle d \rangle = 1$, $A_{ij} = 1$.
 Case 3: $a_{ij} = 7$, $(a_{ij} - 5) = 2$, $(a_{ij} - 6) = 1$, $\langle d \rangle = 1$, $A_{ij} = 1$.
 Case 4: $a_{ij} = 0$, $(a_{ij} - 5) = -5$, $(a_{ij} - 6) = -6$, $\langle d \rangle = 1$, $A_{ij} = 0$.
 Case 5: $a_{ij} = 1$, $(a_{ij} - 5) = -4$, $(a_{ij} - 6) = -5$, $\langle d \rangle = 1$, $A_{ij} = 0$.
 Case 6: $a_{ij} = 2$, $(a_{ij} - 5) = -3$, $(a_{ij} - 6) = -4$, $\langle d \rangle = 1$, $A_{ij} = 0$.
 Case 7: $a_{ij} = 3$, $(a_{ij} - 5) = -2$, $(a_{ij} - 6) = -3$, $\langle d \rangle = 1$, $A_{ij} = 0$.
 Case 8: $a_{ij} = 4$, $(a_{ij} - 5) = -1$, $(a_{ij} - 6) = -2$, $\langle d \rangle = 1$, $A_{ij} = 0$.

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