DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 00-07

Technology Life-Cycles and Endogenous Growth

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August 2000

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Technology Life-Cycles and Endogenous Growth

Abstract

1. Introduction

$$\mu \quad \P_{\circledR}$$

$$t \qquad i \qquad t \quad \frac{\quad t}{t}$$

2.2. Individuals

6

2.3. The Technology and Potential versus Actual Productivity

t t

t

7

t

$$t \quad \frac{t \quad t \quad t}{\bar{\circledR}}$$

t

6

7 Á z

i t

t+1 t i

2.4. Adoption of New Technologies and Next Generation Machines

2

$$\hat{A}_t^j ; z_t^j ; q_t^j ; L_t^j \qquad t \quad t^{j\ \circledR} \quad t^{j\ 1 \,\dot{\imath}\, \circledR} \quad \dot{\imath} \quad t \; t^{j} \; \dot{\imath} \quad t \; t^{j}$$

t t

8 2

$$\begin{matrix} t^{j} \\ \end{matrix} \quad " \quad \mu \quad \dot{\imath} \; \frac{\qquad}{\qquad\qquad\qquad} \quad \P \quad \frac{t^{j} \; t^{j\ \circledR} \quad t^{j\ 1 \,\dot{\imath}\, \circledR}}{\qquad\qquad\qquad\qquad\qquad} \quad \#_{\dot{\imath}\, \circledR}$$

i

t

Lemma 1: $\frac{@q_t^j}{@(t \,\dot{\imath}\, v)}$ $\frac{@q_t^j}{@x_t}$

Proof:

$$\frac{t^{j}}{t} \qquad \frac{\qquad}{\dot{\imath}} \; \frac{t^{j}}{\dot{\imath}^{\,2}}$$

$$\frac{t^{j}}{\dot{\imath}} \qquad \frac{\qquad}{\dot{\imath}} \; \frac{t^{j}}{t^{2}}$$

$$\dot{A}_t \quad \_ \quad \circledR \quad i \frac{t\ i}{t} \quad i \frac{i}{i} \quad ,$$

Proof:

i

Lemma 3: $8_{t\ ,} \ \acute{} \quad i \quad - i$

$$\frac{x_t}{i} \qquad \frac{x_t}{t}$$

$8_{t\ ,\ t} \acute{} \ f \quad i \quad i \quad i \quad i \quad i \quad g$

$$\frac{\dot{A}_t}{i} \qquad \frac{\dot{A}_t}{t} \qquad \frac{\dot{A}_t}{}$$

Proof: 8 t , x t ³ ®

exp

Lemma 4: $8_{t\,,}$

$$\frac{x_t}{x_t}$$

$8_{t\,,\ t}$

$$\frac{Á_t}{Á_t}$$

Proof: $x_t \quad x_t \ i \ x_t \quad 8_{t\,,}$

$$\frac{x_t}{x_t} \quad i \quad \frac{x_t}{i \ x_t}^{3} \ x_{t;v} \ i \ -'$$

$$Á_t \quad Á_t \ i \ Á_t \quad 8_{t\,,\ t;v}$$

$$\frac{Á_t}{Á_t} \quad i \quad \frac{Á_t}{i \ Á_t}^{3} \ Á_t \ i \ -'$$

¤

2.5. Equilibrium R&D E®ort in Inventions versus Innovations

10 $Á_t$

x_t

$Á_t \quad x_t$

µ ¤ ¶° ¤ $\frac{t}{¤_t}$ ¤ .

10 8)8)8 ; 8

$^{¤}_{t}$

.

$^{¤;n}_{t}$ $^{¤}_{t}$

$$!^{Á;n}_{t},\ !^{x;n}_{t} \quad \Big(\ ^{Á}_{t} \ \mathrm{i} \ ^{x}_{t} \ ^{Á}_{t} \frac{^{Á;n}_{t}}{^{Á}_{t}} \quad ^{x}_{t} \ \mathrm{i} \ ^{Á}_{t} \ ^{x}_{t} \frac{^{x;n}_{t}}{^{z}_{t}} \ \mathrm{i} \quad ^{Á;n}_{t} \quad ^{x;n}_{t} \ \Big)$$

$$^{Á}_{t} \ \mathrm{i} \ ^{x}_{t} \ ^{Á}_{t} \ ^{Á;n}_{t} \ ^{Á}_{t} \quad ^{x}_{t} \ \mathrm{i} \ ^{Á}_{t} \ ^{x}_{t} \ ^{x;n}_{t} \ ^{x}_{t}$$

11 $\quad ^{¤}_{t} \quad ^{¤}_{t} \ \mathrm{i} \ ^{¤}_{t} \quad \mathbf{R}^{1}_{0} \ ^{¤;j}_{t}$

$^{Á}_{t}$

Á v Á
t t t

x Á x
t t t

Proposition 1:

8

Á
Á;n t i x Á
t t t

x
x;n t i Á x
t t t

Á;n x;n Á Á;n Á;n
t t t t t

x x;n x;n
t t t

Proof:

¤

¤ ¤ ¤;n ¤
t t t

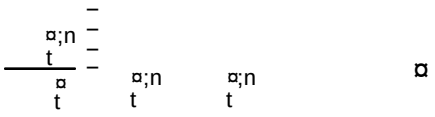
12 13

12

! ¤ I

13

Proposition 2: 8



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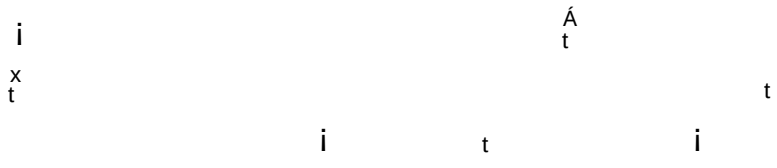


Á

x

0

Á x

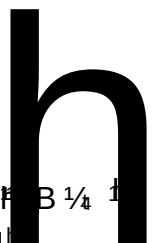
i

t i

6. Appendix

2

¤ ¤ h
t t i Á (1 ¡ l x t)



B ¼ t 1) t91 08Tj 5.2522.25 .31 0.11 Tf 00.132 Tc (s) Tj 5.25+0 TD31F0 12 Tf 0 -0.0533 Tc (l) Tj 4.5 3 TD /F12 0.11 Tf 0.318 Tc (x054) Tj -0.75 -8.25

$$8_{t} \cdot \quad i$$

$$\frac{\acute{A}_{t}\ x_{t}}{i} \qquad \frac{\acute{A}_{t}\ x_{t}}{t}$$

$$8_{t} \qquad i \quad \frac{@(I_{t}^{\acute{A}}=I_{t}^{x})}{@(t\ ;\ v)} \qquad \frac{@(I_{t}^{\acute{A}}=I_{t}^{x})}{@x_{t}} \ ,$$

□

Figure 1

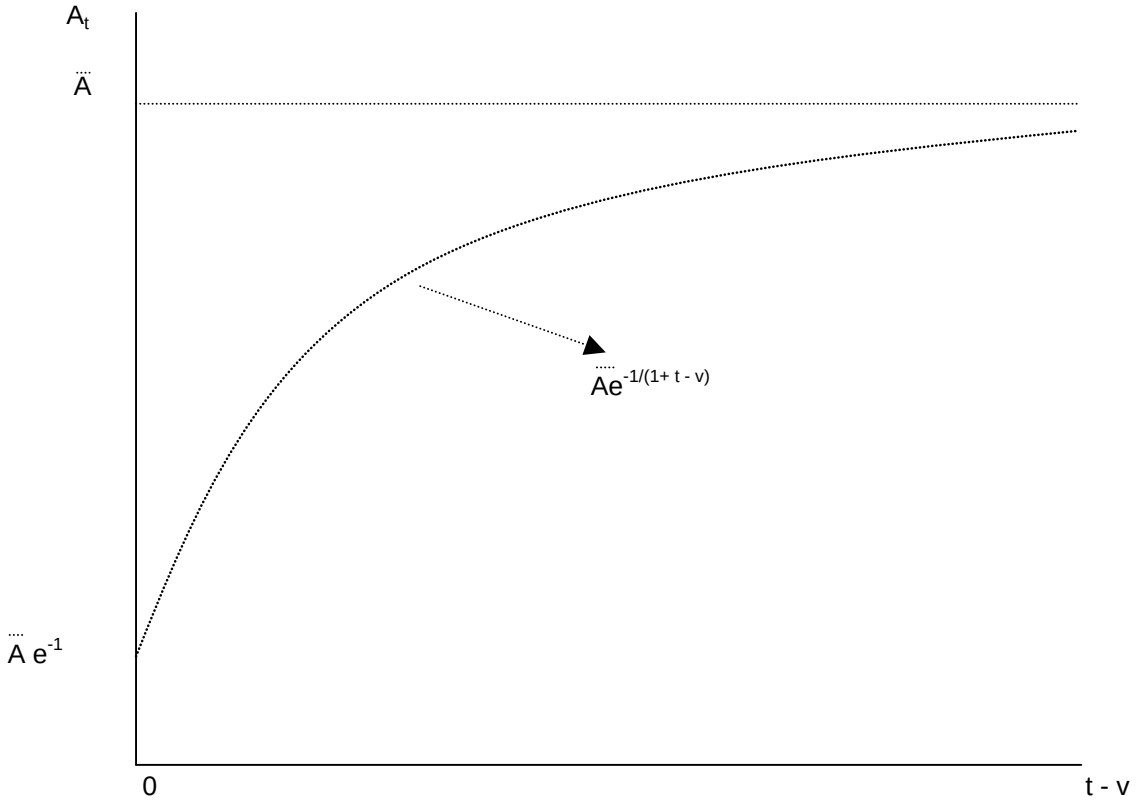


Figure 2

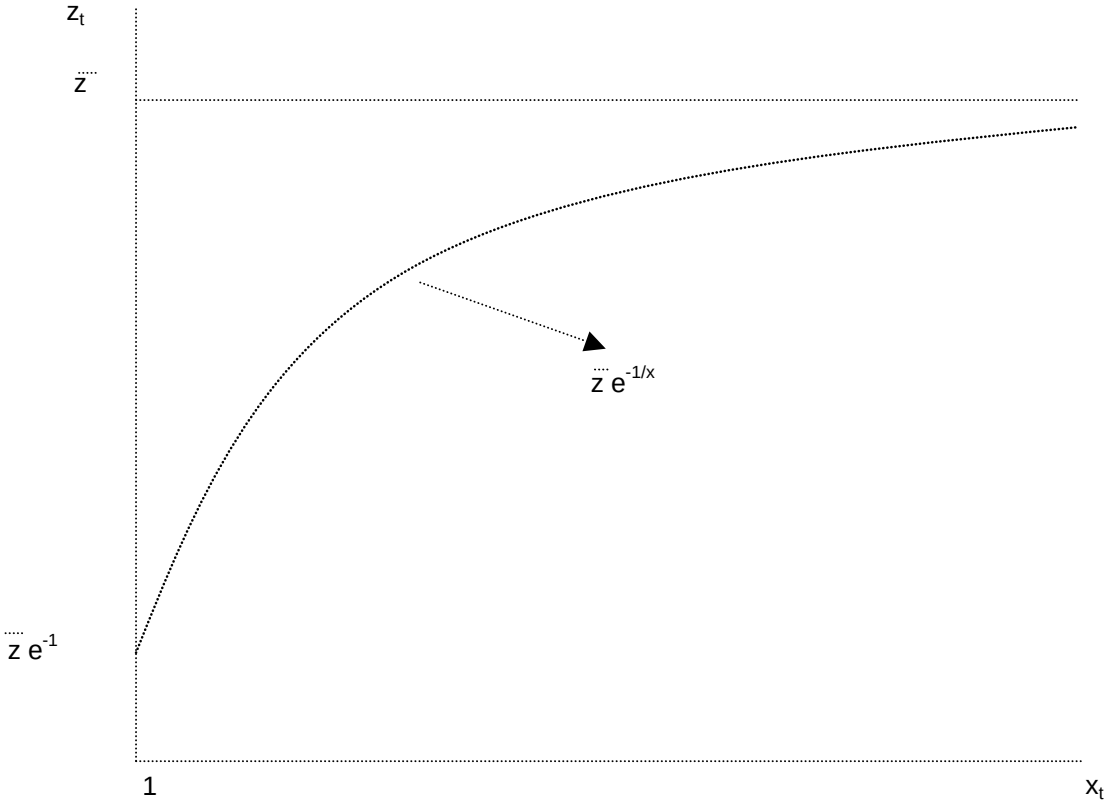


Figure 3

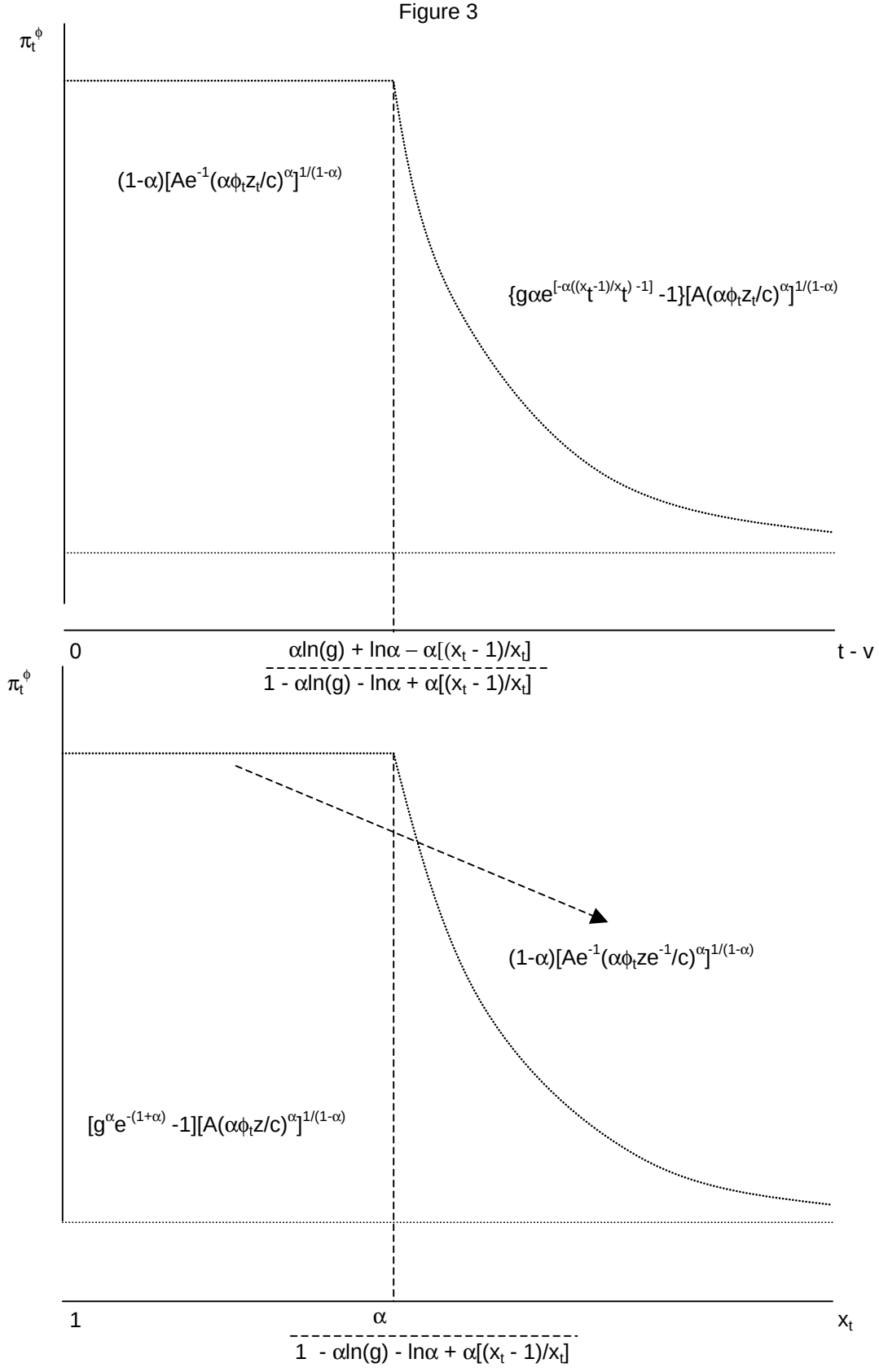
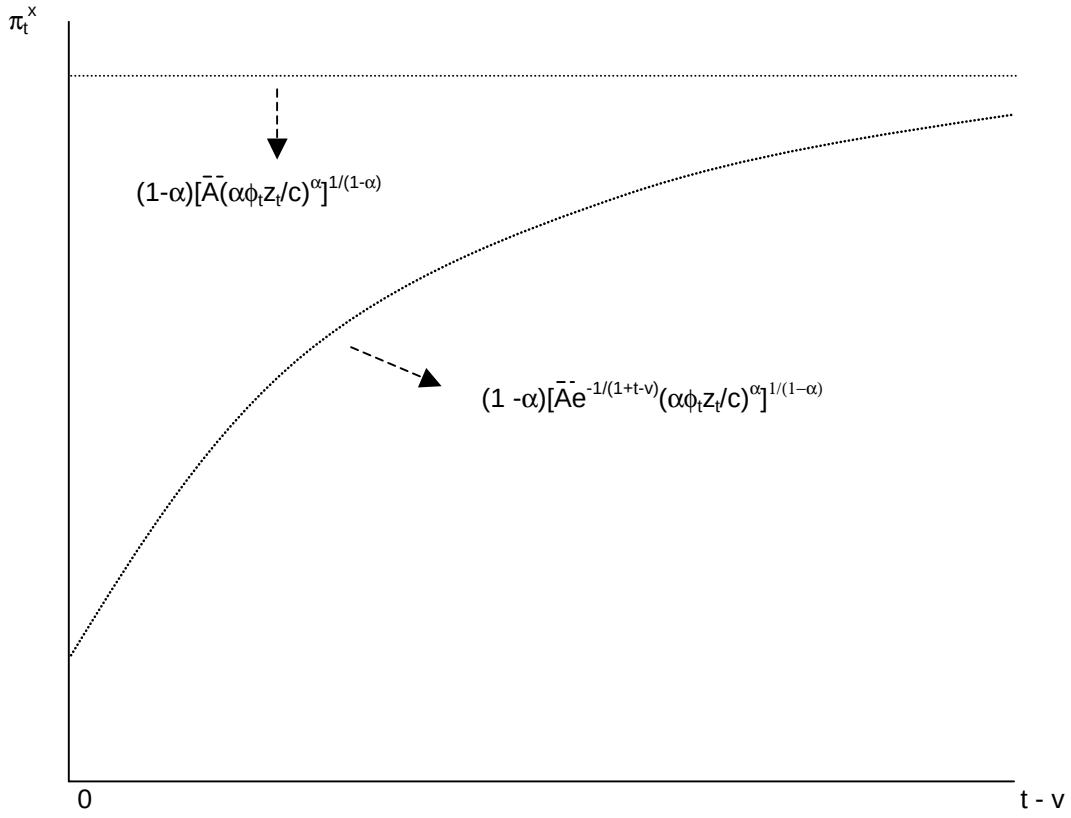

π_t^{ϕ}
$(1-\alpha)[Ae^{-1}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$\{g\alpha e^{[-\alpha((x_t-1)/x_t)-1]}-1\}[A(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
0
$\frac{\alpha\ln(g)+\ln\alpha-\alpha[(x_t-1)/x_t]}{1-\alpha\ln(g)-\ln\alpha+\alpha[(x_t-1)/x_t]}$
t - v
π_t^{ϕ}
$(1-\alpha)[Ae^{-1}(\alpha\phi_t z e^{-1}/c)^{\alpha}]^{1/(1-\alpha)}$
$[g^{\alpha}e^{-(1+\alpha)}-1][A(\alpha\phi_t z/c)^{\alpha}]^{1/(1-\alpha)}$
1
$\frac{\alpha}{1-\alpha\ln(g)-\ln\alpha+\alpha[(x_t-1)/x_t]}$
x_t

Figure 4


π_t^x
$(1-\alpha)[\bar{\bar{A}}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$(1-\alpha)[\bar{\bar{A}}e^{-1/(1+t-v)}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
0
t - v

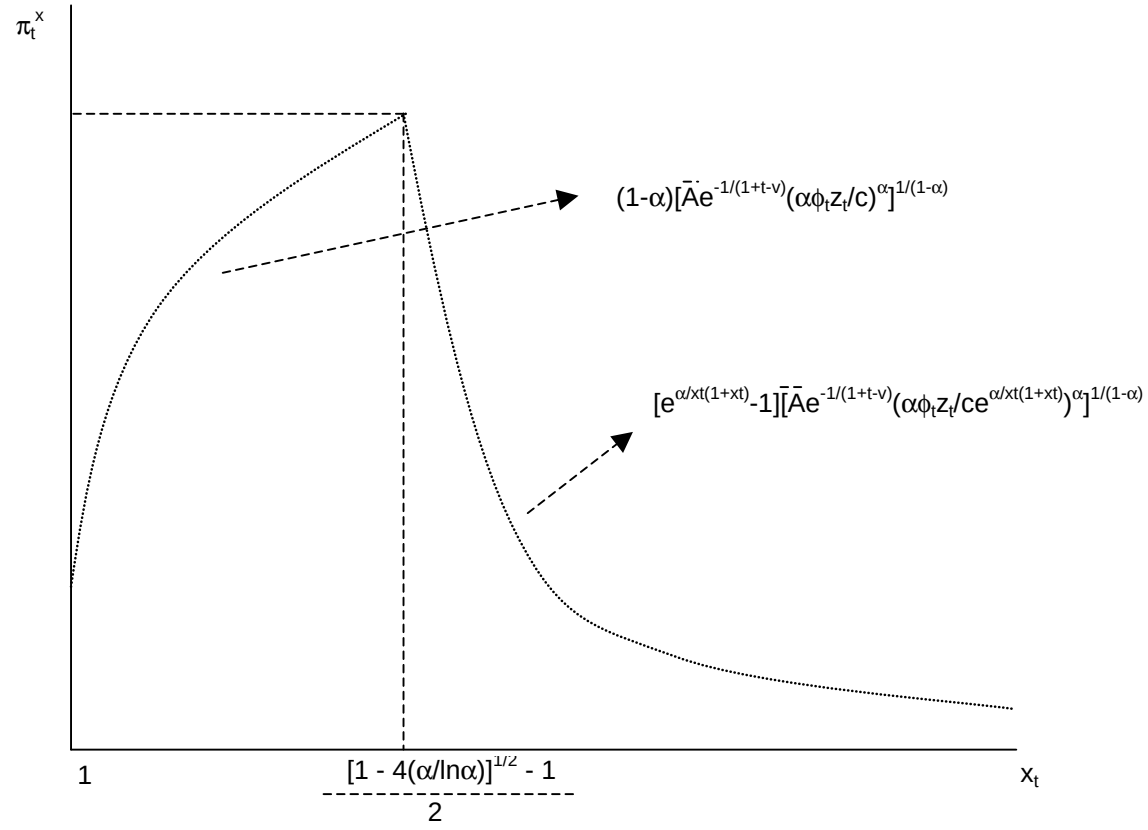

π_t^x
$(1-\alpha)[\bar{\bar{A}}e^{-1/(1+t-v)}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$[e^{\alpha/xt(1+xt)}-1][\bar{\bar{A}}e^{-1/(1+t-v)}(\alpha\phi_t z_t/ce^{\alpha/xt(1+xt)})^{\alpha}]^{1/(1-\alpha)}$
1
$\frac{[1-4(\alpha/\ln\alpha)]^{1/2}-1}{2}$
x_t

Figure 5:

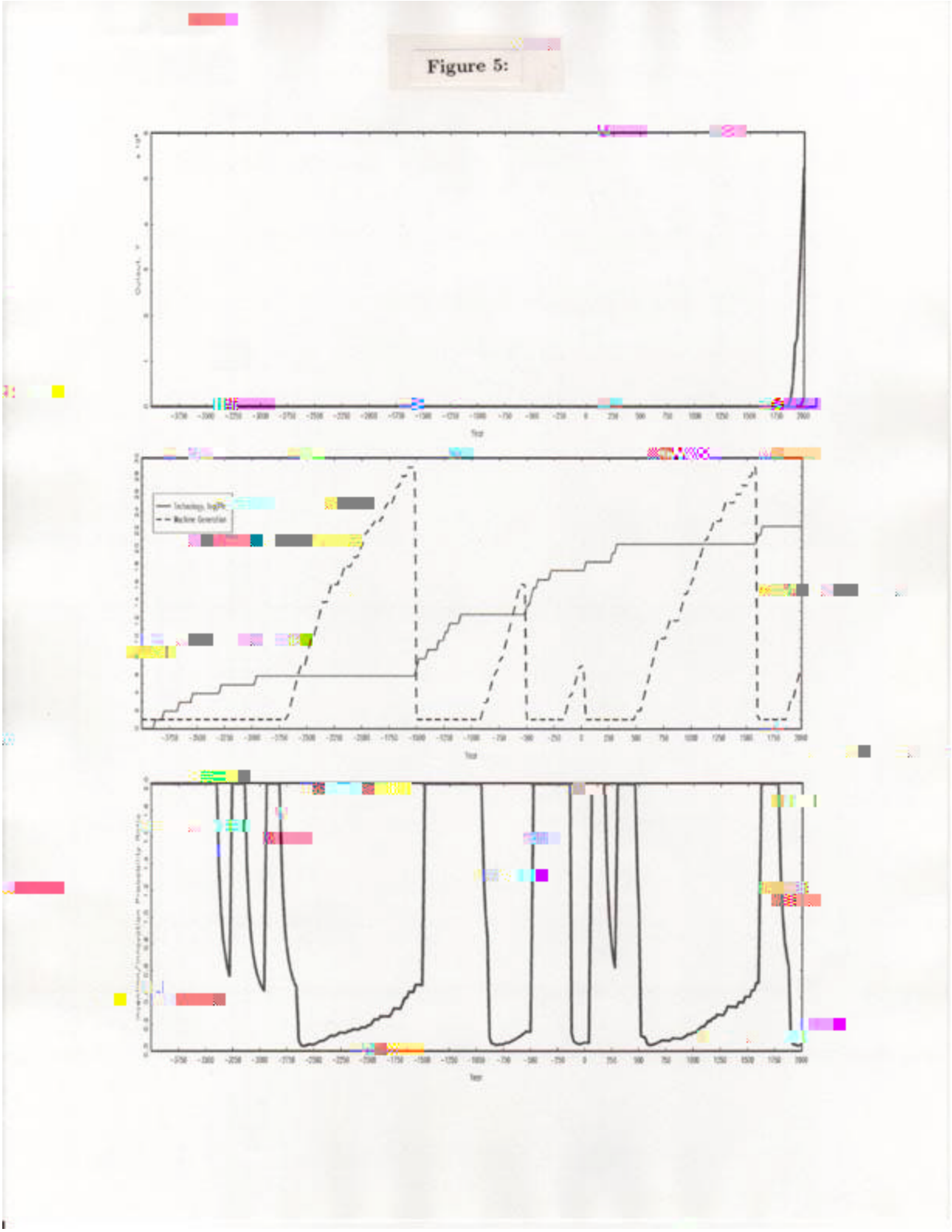


Figure 6:

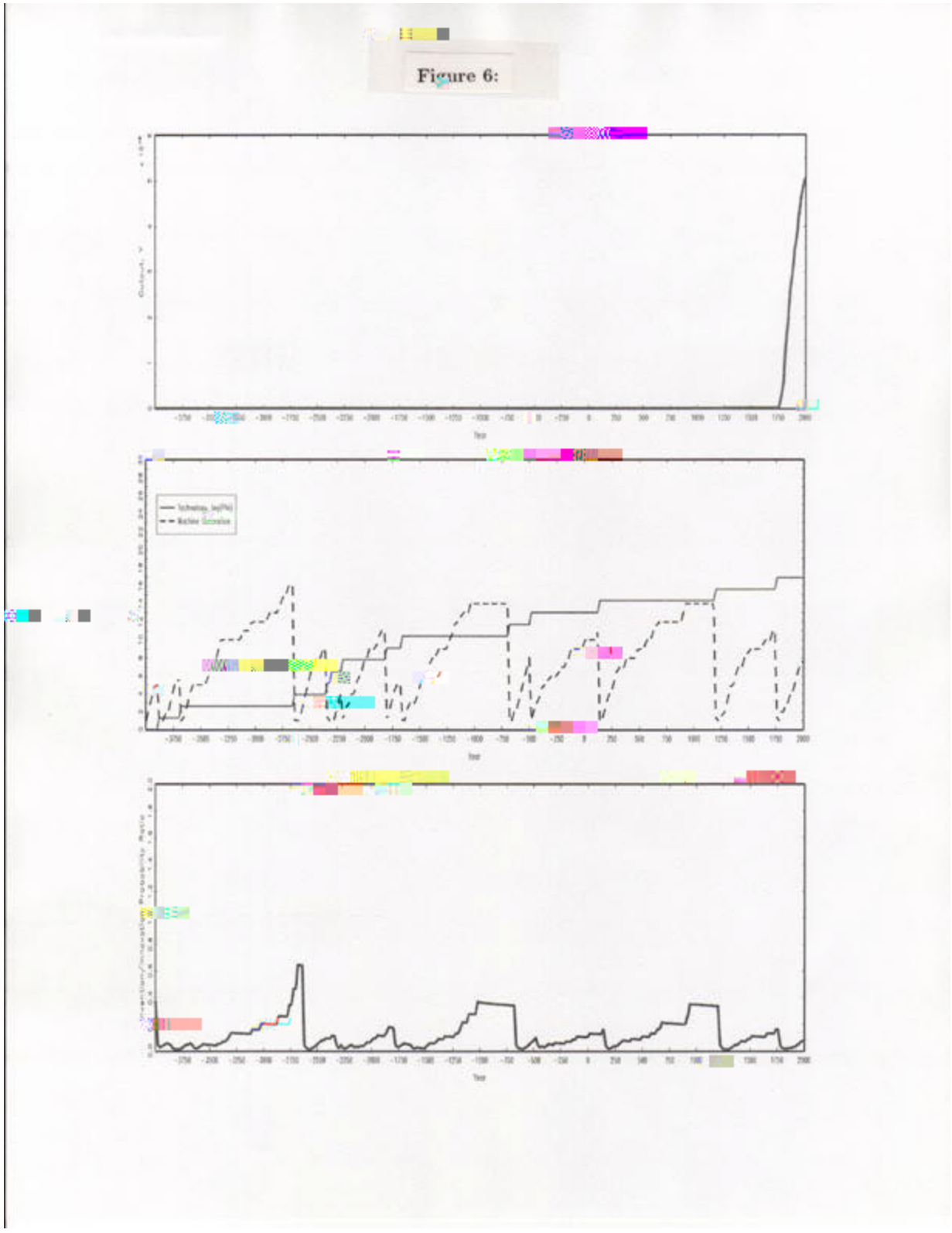


Table 1:

Á		
x		
0	¡ 20	¡ 20